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ON SPHERICALLY SYMMETRIC SOLUTIONS WITH HORIZON IN MODEL WITH MULTICOMPONENT ANISOTROPIC FLUID

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Abstract

A family of spherically symmetric solutions in the model with m-component multicomponent anisotropic fluid is considered. The metric of the solution depends on parameters $q_s > 0$, $s = 1, \dots, m$, relating radial pressures and the densities and contains $(n-1)m$ parameters corresponding to Ricci-flat "internal space" metrics and obeying certain $m(m-1)/2$ ("orthogonality") relations. For $q_s = 1$ (for all s) and certain equations of state ($p_i^s = \pm\rho^s$) the metric coincides with the metric of intersecting black brane solution in the model with antisymmetric forms. A family of solutions with (regular) horizon corresponding to natural numbers $q_s = 1, 2, \dots$ is singled out. Certain examples of "generalized simulation" of intersecting M-branes in $D = 11$ supergravity are considered. The post-Newtonian parameters β and γ corresponding to the 4-dimensional section of the metric are calculated.

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1 Introduction

This paper is devoted to spherically-symmetric solutions with a horizon in the multidimensional model with multicomponent anisotropic fluid defined on product manifolds $\mathbf{R} \times M_0 \times \ldots \times M_n$. These solutions in certain cases may simulate black brane solutions [1, 2, 3] (for a review on p-brane solutions see [4] and references therein).

We remind that p-brane solutions (e.g. black brane ones) usually appear in the models with antisymmetric forms and scalar fields (see also [5]-[15]). Cosmological and spherically symmetric solutions with p-branes are usually obtained by the reduction of the field equations to the Lagrange equations corresponding to Toda-like systems [14]. An analogous reduction for the models with multicomponent "perfect" fluid was done earlier in [18, 19].

For cosmological models with antisymmetric forms without scalar fields any p-brane is equivalent to an multicomponent anisotropic perfect fluid with the equations of state:

$$p_i = -\rho, \quad \text{or} \quad p_i = \rho, \tag{1.1}$$

when the manifold M_i belongs or does not belong to the brane world-volume, respectively (here p_i is the pressure in M_i and ρ is the density, see Section 2).

In this paper we find a new family of exact spherically-symmetric solutions in the model with m-component anisotropic fluid for the following equations of state (see Appendix for more familiar form of eqs. of state):

$$p_r^s = -\rho^s (2q_s - 1)^{-1}, \qquad p_0^s = \rho^s (2q_s - 1)^{-1}, \tag{1.2}$$

and

$$p_i^s = \left(1 - \frac{2U_i^s}{d_i}\right) \rho^s / (2q_s - 1), \tag{1.3}$$

$i > 1$, $s = 1, \ldots, m$, where for s-th component: ρ^s is a density, p_r^s is a radial pressure, p_i^s is a pressure in M_i, $i = 2, \ldots, n$. Here parameters U_i^s $(i > 1)$ and the parameters $q_s = U_1^s > 0$ obey the following "orthogonality" relations (see also Section 2 below)

$$B_{sl} = 0, \qquad s \neq l \tag{1.4}$$

where

$$B_{sl} \equiv \sum_{i=1}^{n} \frac{U_i^s U_i^l}{d_i} + \frac{1}{2 - D} \left(\sum_{i=1}^{n} U_i^s\right) \left(\sum_{j=1}^{n} U_j^l\right), \tag{1.5}$$

$q_s \neq 1/2$; and $s, l = 1, \ldots, m$. The manifold M_0 is d_0-dimensional sphere in our case and p_0^s is the pressure in the tangent direction.

The one-component case was considered earlier in [1]. For special case with $q_s = 1$ see [2] and [3] (for one-component and multicomponent case, respectively).

The paper is organized as follows. In Section 2 the model with multicomponent (anisotropic or "perfect") fluid is formulated. In Section 3 a subclass of spherically symmetric solutions (generalizing solutions from [3]) is presented and solutions with (regular) horizon corresponding to integer q_s are singled out. Section 4 deals with certain examples of two-component solutions in dimension $D = 11$ containing for $q_s = 1$ intersecting $M2 \cap M2$, $M2 \cap M5$ and $M5 \cap M5$ black brane metrics. In Section 5 the post-Newtonian parameters for the 4-dimensional section of the metric are calculated. In the Appendix a class of general spherically symmetric solutions in the model under consideration is presented.

2 The model

Here, we consider a family of spherically symmetric solutions to Einstein equations with an multicomponent anisotropic fluid matter source

$$R_N^M - \frac{1}{2}\delta_N^M R = kT_N^M \qquad (2.1)$$

defined on the manifold

$$M = \underset{\substack{radial\\variable}}{\mathbf{R}_{.}} \times \underset{\substack{spherical\\variables}}{(M_0 = S^{d_0})} \times \underset{time}{(M_1 = \mathbf{R})} \times M_2 \times \ldots \times M_n, \qquad (2.2)$$

with the block-diagonal metrics

$$ds^2 = e^{2\gamma(u)} du^2 + \sum_{i=0}^{n} e^{2X^i(u)} h^{(i)}_{m_i n_i} dy^{m_i} dy^{n_i}. \qquad (2.3)$$

Here $\mathbf{R}_{.} = (a, b)$ is interval. The manifold M_i with the metric $h^{(i)}$, $i = 1, 2, \ldots, n$, is the Ricci-flat space of dimension d_i:

$$R_{m_i n_i}[h^{(i)}] = 0, \qquad (2.4)$$

and $h^{(0)}$ is standard metric on the unit sphere S^{d_0}

$$R_{m_0 n_0}[h^{(0)}] = (d_0 - 1) h^{(0)}_{m_0 n_0}, \tag{2.5}$$

u is radial variable, κ is the multidimensional gravitational constant, $d_1 = 1$ and $h^{(1)} = -dt \otimes dt$.

The energy-momentum tensor is adopted in the following form

$$T^M_N = \sum_{s=1}^{m} T^{(s)M}_N, \tag{2.6}$$

where

$$T^{(s)M}_N = \mathrm{diag}(-(2q_s - 1)^{-1}\rho^s, (2q_s - 1)^{-1}\rho^s \delta^{m_0}_{k_0}, -\rho^s, p^s_2 \delta^{m_2}_{k_2}, \dots, p^s_n \delta^{m_n}_{k_n}), \tag{2.7}$$

$q_s > 0$ and $q_s \neq 1/2$. The pressures p^s_i and the density ρ^s obeys the relations (1.3) with constants U^s_i, $i > 1$.

The "conservation law" equations

$$\nabla_M T^{(s)M}_N = 0 \tag{2.8}$$

are assumed to be valid for all s.

In what follows we put $\kappa = 1$ for simplicity.

3 Exact solutions

Let us define

$$1^o. \quad U^s_0 = 0, \tag{3.1}$$

$$2^o. \quad U^s_1 = q_s, \tag{3.2}$$

$$3^o. \quad (U^s, U^l) = U^s_i G^{ij} U^l_j \tag{3.3}$$

where $U^s = (U^s_i)$ is $(n+1)$-dimensional vector and

$$G^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2 - D} \tag{3.4}$$

are components of the matrix inverse to the matrix of the minisuperspace metric [16, 17]

$$(G_{ij}) = (d_i \delta_{ij} - d_i d_j), \tag{3.5}$$

$i, j = 0, \dots, n$, and $D = 1 + \sum\limits_{i=0}^{n} d_i$ is the total dimension.

In our case the scalar products (3.3) are given by relations:

$$(U^s, U^l) = B_{kl} \tag{3.6}$$

with B_{kl} from (1.5) and hence due to (1.4) vectors U^s are mutually orthogonal, i.e.

$$(U^s, U^l) = 0, \qquad s \neq l. \tag{3.7}$$

It is proved in Appendix that the relation 1^o implies

$$(U^s, U^s) > 0, \tag{3.8}$$

for all s.

For the equations of state (1.2) and (1.3) with parameters obeying (1.4) we have obtained the following spherically symmetric solutions to the Einstein equations (2.1) (see Appendix)

$$ds^2 = J_0 \left(\frac{dr^2}{1 - \frac{2\mu}{r^d}} + r^2 d\Omega^2_{d_0} \right) - J_1 \left(1 - \frac{2\mu}{r^d} \right) dt^2 \\ + \sum_{i=2}^{n} J_i h^{(i)}_{m_i n_i} dy^{m_i} dy^{n_i}, \tag{3.9}$$

$$\rho^s = \frac{(2q_s - 1)(dq_s)^2 P_s (P_s + 2\mu)(1 - 2\mu r^{-d})^{q_s - 1}}{2(U^s, U^s)(\prod_{s=1}^{m} H_s)^2 J_0 r^{2d_0}}, \tag{3.10}$$

by methods similar to obtaining p-brane solutions [14]. Here $d = d_0 - 1$, $d\Omega^2_{d_0} = h^{(0)}_{m_0 n_0} dy^{m_0} dy^{n_0}$ is spherical element, the metric factors

$$J_i = \prod_{s=1}^{m} H_s^{-2U^{si}/(U^s, U^s)}, \tag{3.11}$$

$$H_s = 1 + \frac{P_s}{2\mu} \left[1 - \left(1 - \frac{2\mu}{r^d} \right)^{q_s} \right]; \tag{3.12}$$

$P > 0$, $\mu > 0$ are constants and

$$U^{si} = G^{ij} U^s_j = \frac{U^s_i}{d_i} + \frac{1}{2 - D} \sum_{j=0}^{n} U^s_j. \tag{3.13}$$

Using (3.13) and $U_0^s = 0$ we get

$$U^{s0} = \frac{1}{2-D} \sum_{j=0}^{n} U_j^s \tag{3.14}$$

and hence one can rewrite (3.9) as follows

$$ds^2 = J_0 \Bigg[\frac{dr^2}{1 - \frac{2\mu}{r^d}} + r^2 d\Omega_{d_0}^2 - \left(\prod_{s=1}^{m} H_s^{-2q_s/(U^s,U^s)} \right) \left(1 - \frac{2\mu}{r^d} \right) dt^2 + \tag{3.15}$$
$$+ \sum_{i=2}^{n} \left(\prod_{s=1}^{m} H_s^{-2U_i^s/(d_i(U^s,U^s))} \right) h^{(i)}_{m_i n_i} dy^{m_i} dy^{n_i} \Bigg].$$

These solutions are special case of general solutions spherically symmetric solutions obtained in Appendix by method suggested in [19].

Black holes for natural q_s.

For natural

$$q_s = 1, 2, \ldots, \tag{3.16}$$

the metric has a horizon at $r^d = 2\mu = r_h^2$. Indeed, for these values of q_s the functions $H_s(r) > 0$ are smooth in the interval $(r_*, +\infty)$ for some $r_* < r_h$. For odd $q_s = 2m_s + 1$ (for all s) one get $r_* = 0$.

A global structure of the black hole solution corresponding to these values of q_s will be a subject of a separate publication.

It was shown in [1] that in one-component case for $2U^{s0} \neq -1$ and $0 < q_s < 1$ one get singularity at $r^d \to 2\mu$.

Remark. For non-integer $q_s > 1$ the function $H_s(r)$ have a non-analytical behavior in the vicinity of $r^d = 2\mu$. In this case one may conject that the limit $r^d \to 2\mu$ corresponds to the singularity (in general case) but here a separate investigation is needed.

4 Examples: generalized simulation of intersecting black branes

The solutions with a horizon from the previous section allow us to simulate the intersecting black brane solutions [4] in the model with antisymmetric forms without scalar fields [2] when all $q_s = 1$.

These solutions may be also generalized to the case of general natural $q_s \in \mathbf{N}$. In this case the parameters U_i^s and the pressures have the following form

$$
\begin{array}{rclrcl}
U_i^s & = & q_s d_i, & p_i^s & = & -\rho^s, \quad i \in I_s; \\
 & & 0, & & & (2q_s - 1)^{-1}\rho^s, \quad i \notin I_s.
\end{array}
\tag{4.1}
$$

Here $I_s = \{i_1, \ldots, i_k\} \in \{1, \ldots n\}$ is the index set [4] corresponding to "brane" submanifold $M_{i_1} \times \ldots \times M_{i_k}$.

The "orthogonality" relations (1.4) lead us to the following dimension of intersection of brane submanifolds [4]

$$
d(I_s \cap I_l) = \frac{d(I_s)d(I_l)}{D-2}, \qquad s \neq l,
\tag{4.2}
$$

where $d(I_s) = \sum_{i \in I_s} d_i$ is dimension of p-brane worldvolume.

Remark. The set of Diophantus equations (4.2) was solved explicitly in [20] for so-called "flower" Ansatz from [21]. The solution in this case takes place for infinite number of dimensions $D = 6, 10, 11, 14, 18, 20, 26, 27, ...$ etc.

As an example, here we consider a "generalized simulation" of intersecting $M2 \cap M5$, $M2 \cap M2$ and $M5 \cap M5$ black branes in $D = 11$ supergravity. In what follows functions H_s, $s = 1, 2$, are defined in (3.12).

a). For an analog of intersecting $M2 \cap M5$ branes the metric reads:

$$
\begin{aligned}
ds^2 = H_1^{1/(3q_1)} H_2^{2/(3q_2)} \Bigg[& \frac{dr^2}{1 - 2\mu/r^d} + r^2 d\Omega_{d_0}^2 \\
& - H_1^{-1/q_1} H_2^{-1/q_2} \left\{ \left(1 - \frac{2\mu}{r^d}\right) dt^2 + dy^{m_2} dy^{m_2} \right\} \\
& + H_2^{-1/q_2} h^{(3)}_{m_3 n_3} dy^{m_3} dy^{n_3} + H_1^{-1/q_1} dy^{m_4} dy^{m_4} + h^{(5)}_{m_5 n_5} dy^{m_5} dy^{n_5} \Bigg],
\end{aligned}
\tag{4.3}
$$

where $M2$-brane includes three one-dimensional spaces: M_2, M_4 and the time manifold M_1; and $M5$-brane includes M_1,M_2 and M_3 ($d_3 = 4$).

b). An analog of two electrical $M2$ branes intersecting on the time manifold has the following metric

$$
\begin{aligned}
ds^2 = H_1^{1/(3q_1)} H_2^{1/(3q_2)} \Bigg[& \frac{dr^2}{1 - 2\mu/r^d} + r^2 d\Omega_{d_0}^2 \\
& - H_1^{-1/q_1} H_2^{-1/q_2} \left(1 - \frac{2\mu}{r^d}\right) dt^2
\end{aligned}
\tag{4.4}
$$

$$+H_1^{-1/q_1} h^{(2)}_{m_2 n_2} dy^{m_2} dy^{n_2} + H_2^{-1/q_2} h^{(3)}_{m_3 n_3} dy^{m_3} dy^{n_3} + h^{(4)}_{m_4 n_4} dy^{m_4} dy^{n_4} \Bigg],$$

where $d_2 = d_3 = 2$.

c). For an analog of two intersecting $M5$ branes the dimension of intersection is 4 and the metric reads

$$\begin{aligned} ds^2 = H_1^{2/(3q_1)} H_2^{2/(3q_2)} \Bigg[\frac{dr^2}{1 - 2\mu/r} + r^2 d\Omega_2^2 \\ -H_1^{-1/q_1} H_2^{-1/q_2} \left\{ \left(1 - \frac{2\mu}{r}\right) dt^2 + h^{(2)}_{m_2 n_2} dy^{m_2} dy^{n_2} \right\} \\ +H_1^{-1/q_1} h^{(3)}_{m_3 n_3} dy^{m_3} dy^{n_3} + H_2^{-1/q_2} h^{(4)}_{m_4 n_4} dy^{m_4} dy^{n_4} \Bigg]. \end{aligned} \tag{4.5}$$

Here $d_0 = d_3 = d_4 = 2$ and $d_2 = 3$.

For the density of s-th component we get in any of these three cases

$$\rho^s = \frac{(2q_s - 1) d^2 P_s (P_s + 2\mu)(1 - 2\mu r^{-d})^{q_s - 1}}{4 (H_1 H_2)^2 J_0 r^{2d_0}}, \tag{4.6}$$

where

$$J_0 = \prod_{s=1}^{2} H_s^{d(I_s)/(9q_s)} \tag{4.7}$$

and $d(I_s) = 3, 6$ for $M2, M5$ branes, respectively.

5 Physical parameters

5.1 Gravitational mass and PPN parameters

Here we put $d_0 = 2$ $(d = 1)$. Let us consider the 4-dimensional space-time section of the metric (3.15). Introducing a new radial variable by the relation:

$$r = R \left(1 + \frac{\mu}{2R}\right)^2, \tag{5.1}$$

we rewrite the 4-section in the following form:

$$\begin{aligned} ds^2_{(4)} = \left(\prod_{s=1}^{m} H^{-2U^{s0}/(U^s,U^s)}\right) \Bigg[- \left(\prod_{s=1}^{m} H_s^{-2q_s/(U^s,U^s)}\right) \left(\frac{1 - \frac{\mu}{2R}}{1 + \frac{\mu}{2R}}\right)^2 dt^2 \\ + \left(1 + \frac{\mu}{2R}\right)^4 \delta_{ij} dx^i dx^j \Bigg] \end{aligned} \tag{5.2}$$

$i,j = 1,2,3$. Here $R^2 = \delta_{ij} x^i x^j$.

The parameterized post-Newtonian (Eddington) parameters are defined by the well-known relations

$$g_{00}^{(4)} = -(1 - 2V + 2\beta V^2) + O(V^3), \tag{5.3}$$
$$g_{ij}^{(4)} = \delta_{ij}(1 + 2\gamma V) + O(V^2), \tag{5.4}$$

$i,j = 1,2,3$. Here

$$V = \frac{GM}{R} \tag{5.5}$$

is the Newtonian potential, M is the gravitational mass and G is the gravitational constant.

From (5.2)-(5.4) we obtain:

$$GM = \mu + \sum_{s=1}^{m} \frac{P_s q_s (q_s + U^{s0})}{(U^s, U^s)} \tag{5.6}$$

and

$$\beta - 1 = \sum_{s=1}^{m} \frac{|A_s|}{(GM)^2}(q_s + U^{s0}), \tag{5.7}$$
$$\gamma - 1 = -\sum_{s=1}^{m} \frac{P_s q_s}{(U^s, U^s) GM}(q_s + 2U^{s0}), \tag{5.8}$$

where

$$|A_s| = \frac{1}{2} q_s^2 P_s (P_s + 2\mu)/(U^s, U^s) \tag{5.9}$$

(see Appendix) or, equivalently,

$$P_s = -\mu + \sqrt{\mu^2 + 2|A_s|(U^s, U^s) q_s^{-2}} > 0. \tag{5.10}$$

For fixed U_i^s the parameter $\beta - 1$ is proportional to the ratio of two quantities: the weighted sum of multicomponent anisotropic fluid density parameters $|A_s|$ and the gravitational radius squared $(GM)^2$.

5.2 Hawking temperature

The Hawking temperature of the black hole may be calculated using the well-known relation [22]

$$T_H = \frac{1}{4\pi\sqrt{-g_{tt}g_{rr}}} \frac{d(-g_{tt})}{dr}\bigg|_{horizon} . \tag{5.11}$$

We get

$$T_H = \frac{d}{4\pi(2\mu)^{1/d}} \prod_{s=1}^{m} \left(1 + \frac{P_s}{2\mu}\right)^{-q_s/(U^s,U^s)} . \tag{5.12}$$

Here all q_s are natural numbers.

For any of $D = 11$ metrics from Section 4 the Hawking temperature reads $T_H = \frac{d}{4\pi(2\mu)^{1/d}} \prod_{s=1}^{2} \left(1 + \frac{P_s}{2\mu}\right)^{-1/(2q_s)}$.

6 Conclusions

In this paper, using the methods developed earlier for obtaining perfect fluid and p-brane solutions, we have considered a family of spherically symmetric solutions in the model with m-component anisotropic fluid when the equations of state (1.2)- (1.4) are imposed. The metric of any solution contains $(n-1)$ Ricci-flat "internal" space metrics and depends upon a set of parameters U_i^s, $i > 1$.

For $q_s = 1$ (for all s) and certain equations of state (with $p_i^s = \pm\rho^s$) the metric of the solution coincides with that of intersecting black brane solution in the model with antisymmetric forms without dilatons [3]. For natural numbers $q_s = 1, 2, \ldots$ we have obtained a family of solutions with regular horizon.

Here we have considered three examples of solutions with horizon, that simulate (by fluids) binary intersecting $M2$ and $M5$ black branes in $D = 11$ supergravity.

We have also calculated (for possible estimations of observable effects of extra dimensions) the post-Newtonian parameters β and γ corresponding to the 4-dimensional section of the metric and the Hawking temperature as well.

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Appendix

A Lagrange representation

It is more convenient for finding of exact solutions, to write the stress-energy tensor in cosmological-type form

$$(T^{(s)M}_N) = \mathrm{diag}(-\hat{\rho}^s, \hat{p}^s_0 \delta^{m_0}_{k_0}, \hat{p}^s_1 \delta^{m_1}_{k_1}, \ldots, \hat{p}^s_n \delta^{m_n}_{k_n}), \qquad (A.1)$$

where $\hat{\rho}^s$ and $\hat{p}^s_i$ are "effective" density and pressures of s-th component, respectively, depending upon the radial variable u and the physical density ρ^s and pressures p^s_i are related to the effective ("hat") ones by formulas

$$\rho^s = -\hat{p}^s_1, \quad p^s_r = -\hat{\rho}^s, \quad p^s_i = \hat{p}^s_i, \quad (i \neq 1), \qquad (A.2)$$

$s = 1, \ldots, m$.

The equations of state may be written in the following form

$$\hat{p}_i = \left(1 - \frac{2U^s_i}{d_i}\right) \hat{\rho}^s, \qquad (A.3)$$

where U^s_i are constants, $i = 0, 1, \ldots, n$. It follows from (A.2), (A.3) and $U^s_1 = q_s$ that

$$\rho^s = (2q_s - 1)\hat{\rho}^s. \qquad (A.4)$$

The "conservation law" equations $\nabla_M T^{(s)M}_N = 0$ may be written, due to relations (2.3) and (A.1) in the following form:

$$\dot{\hat{\rho}}^s + \sum_{i=0}^{n} d_i \dot{X}^i (\hat{\rho}^s + \hat{p}^s_i) = 0. \qquad (A.5)$$

Using the equation of state (A.3) we get

$$\hat{\rho}^s = -A_s e^{2U^s_i X^i - 2\gamma_0}, \qquad (A.6)$$

where $\gamma_0(X) = \sum\limits_{i=0}^{n} d_i X^i$ and A_s are constants.

The Einstein equations (2.1) with the relations (A.3) and (A.6) imposed are equivalent to the Lagrange equations for the Lagrangian

$$L = \frac{1}{2} e^{-\gamma+\gamma_0(X)} G_{ij} \dot{X}^i \dot{X}^j - e^{\gamma-\gamma_0(X)} V, \tag{A.7}$$

where

$$V = \frac{1}{2} d_0(d_0 - 1) \exp(2U_i^0 X^i) + A_s \exp(2U_i^s X^i) \tag{A.8}$$

is the potential and the components of the minisupermetric G_{ij} are defined in (3.5),

$$U_i^0 X^i = -X^0 + \gamma_0(X), \qquad U_i^0 = -\delta_i^0 + d_i, \tag{A.9}$$

$i = 0, \ldots, n$ (for cosmological case see [18, 19]).

For $\gamma = \gamma_0(X)$, i.e. when the harmonic time gauge is considered, we get the set of Lagrange equations for the Lagrangian

$$L = \frac{1}{2} G_{ij} \dot{X}^i \dot{X}^j - V, \tag{A.10}$$

with the zero-energy constraint imposed

$$E = \frac{1}{2} G_{ij} \dot{X}^i \dot{X}^j + V = 0. \tag{A.11}$$

It follows from the restriction $U_0^s = 0$ that

$$(U^0, U^s) \equiv U_i^0 G^{ij} U_j^s = 0. \tag{A.12}$$

Indeed, the contravariant components $U^{0i} = G^{ij} U_j^0$ are the following ones

$$U^{0i} = -\frac{\delta_0^i}{d_0}. \tag{A.13}$$

Then we get $(U^0, U^s) = U^{0i} U_i^s = -U_0^s / d_0 = 0$. In what follows we also use the formula

$$(U^0, U^0) = \frac{1}{d_0} - 1 < 0 \tag{A.14}$$

for $d_0 > 1$.

Now we prove that $(U^s, U^s) > 0$ for all $s > 0$. Indeed, minisupermetric has the signature $(-, +, \ldots, +)$ [16, 17], vector U^0 is time-like and orthogonal to any vector $U^s \neq 0$. Hence any vector U^s is space-like.

B General spherically symmetric solutions

When the orthogonality relations (A.12) and (3.7) are satisfied the Euler-Lagrange equations for the Lagrangian (A.10) with the potential (A.8) have the following solutions (see relations from [19] adopted for our case):

$$X^i(u) = -\sum_{\alpha=0}^{m} \frac{U^{\alpha i}}{(U^\alpha, U^\alpha)} \ln |f_\alpha(u - u_\alpha)| + c^i u + \bar{c}^i, \tag{B.1}$$

where u_α are integration constants; and vectors $c = (c^i)$ and $\bar{c} = (\bar{c}^i)$ are dually-orthogonal to co-vectors $U^\alpha = (U_i^\alpha)$, i.e. they satisfy the linear constraint relations

$$U^0(c) = U_i^0 c^i = -c^0 + \sum_{j=0}^{n} d_j c^j = 0, \tag{B.2}$$

$$U^0(\bar{c}) = U_i^0 \bar{c}^i = -\bar{c}^0 + \sum_{j=0}^{n} d_j \bar{c}^j = 0, \tag{B.3}$$

$$U^s(c) = U_i^s c^i = 0, \tag{B.4}$$

$$U^s(\bar{c}) = U_i^s \bar{c}^i = 0. \tag{B.5}$$

Here

$$f_\alpha(\tau) = \begin{cases} R_\alpha \dfrac{\sinh(\sqrt{C_\alpha}\tau)}{\sqrt{C_\alpha}}, & C_\alpha > 0, \quad \eta_\alpha = +1, \\ R_\alpha \dfrac{\cosh(\sqrt{C_\alpha}\tau)}{\sqrt{C_\alpha}}, & C_\alpha > 0, \quad \eta_\alpha = -1, \\ R_\alpha \dfrac{\sin(\sqrt{|C_\alpha|}\tau)}{\sqrt{|C_\alpha|}}, & C_\alpha < 0, \quad \eta_\alpha = +1, \\ R_\alpha \tau, & C_\alpha = 0, \quad \eta_\alpha = +1, \end{cases} \tag{B.6}$$

$\alpha = 0, \ldots, m$; where $R_0 = d_0 - 1$, $\eta_0 = 1$, $R_s = \sqrt{2|A_s|(U^s, U^s)}$, $\eta_s = -\mathrm{sign} A_s$ $(s = 1, \ldots, m)$.

The zero-energy constraint, corresponding to the solution (B.1) reads

$$E = \frac{1}{2} \sum_{\alpha=0}^{m} \frac{C_\alpha}{(U^\alpha, U^\alpha)} + \frac{1}{2} G_{ij} c^i c^j = 0. \tag{B.7}$$

Special solutions. The (weak) horizon condition (i.e. infinite time of propagation of light for $u \to +\infty$) lead us to the following integration constants

$$
\begin{aligned}
\bar{c}^i &= 0, && \text{(B.8)}\\
c^i &= \bar{\mu} \sum_{\alpha=0}^{m} \frac{U_1^{\alpha} U^{\alpha i}}{(U^{\alpha}, U^{\alpha})} - \bar{\mu}\delta_1^i, && \text{(B.9)}\\
C_{\alpha} &= (U_1^{\alpha})^2 \bar{\mu}^2, && \text{(B.10)}
\end{aligned}
$$

where $\bar{\mu} > 0$, $\alpha = 0, \dots, m$. For analogous choice of parameters in p-brane case see [13, 14, 4].

We also introduce a new radial variable $r = r(u)$ by relations

$$
\exp(-2\bar{\mu}u) = 1 - \frac{2\mu}{r^d}, \quad \mu = \bar{\mu}/d > 0, \quad d = d_0 - 1, \tag{B.11}
$$

and put $u_s < 0$ and $A_s < 0$ for all s and also $u_0 = 0$.

The relations of the Appendix imply the formulae (3.9) and (3.10) for the solution from Section 3 with

$$
H_s = \exp(-\bar{\mu} q_s u) f_s(u - u_s), \qquad A_s = -\frac{(dq_s)^2}{2(U^s, U^s)} P_s(P_s + 2\mu), \tag{B.12}
$$

$P_s > 0$.

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